High-Performance Multivariable Control Strategies for Systems Having Time Delays

A new high-performance multivariable time delay compensator is presented that contains the Ogunnaike-Ray compensator as a special case, reduces to the Smith predictor for a single delay, and approaches the realizable part of the process inverse as a limit. The straightforward design procedure is illustrated with several distillation control examples.

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SCOPE

Multivariable systems having multiple time delays are often difficult to control with conventional single-loop controllers. The single-delay problem was attacked by Smith (1957) through addition of a delay compensator to the system, resulting in a new system that is more easily controlled. In this work a completely general multivariable time delay compensator is described that is capable of extending any of several key features of the Smith predictor to the multivariable case. The precise design depends on the time delay structure of the system and can even include the addition of delays to some of the inputs.

CONCLUSIONS AND SIGNIFICANCE

The multidelay compensator has the attractive feature that both time delay and interaction compensation are achieved within a single design, and conventional controller gains (eg., for a PI controller) can be used to tune for the desired balance between performance and robustness. The compensator has the property that predictions of each output are provided to the feedback controller and this property is preserved even in the case of constraints on the manipulated variables. In the case of right half-plane zeroes, the design is augmented to provide compensation for these as well. As demonstrated in the examples, this structure is capable of effective control even in the case of constrained manipulated variables.

Introduction

The dynamic behavior of many chemical processes can be represented by models consisting of sets of differential equations and time delays. The delays may occur directly because of transportation dead time in flow through a pipe, as in recycle loops, or in composition analysis such as the elution time of an on-line chromatograph. Another source of time delays in the input-output models used for control system design is the approximation of a high-order system with a low-order model combined with a time delay. The dynamic behavior of multistage distillation columns is often modeled in this way. (Wood and Berry, 1973, Ogunnaike and Ray, 1983). Thus process control system design for chemical processes with models having time delays is a frequent practical problem.

The presence of time delays makes controller design more difficult for several reasons. The addition of time delays introduces additional phase lag, which can cause the closed-loop system to become unstable at relatively low controller gains. In addition, the presence of time delays in the modeling equations makes analysis of process dynamics and stability of the system more difficult. For example, in the time domain the presence of time delays means that the system is not described by merely a set of ordinary differential equations, but rather by a more complex set of differential-difference equations.

In order to deal with the delays that arise in dynamic models used for process control and to easily handle the mathematical equations that arise, a general procedure for designing high-performance multivariable time delay compensators has been developed. This new technique can effectively handle a large family of multivariable systems containing different time delays and produce high-performance control systems. Yet, the design method is straightforward to apply. In this paper we first review the relationship of our generalized multidelay compensator (GMDC) to earlier time delay compensation designs, then describe the design procedure, and finally demonstrate the performance on some test problems arising in distillation column control.

Previous Work

The earliest time delay compensator is the single-input, single-output Smith Predictor proposed by Otto Smith in 1957. Since then many workers have proposed alternatives or modifications of the Smith predictor structure, each claiming some advantage in a particular situation (Vit, 1979; Krishman et al., 1980; Cook and Price, 1978; Watanabe and Ito, 1981; Furukawa and Shimura, 1983). For multivariable systems with multiple delays far fewer structures have been proposed. Alevisakis and Seborg (1973, 1974) have proposed a multivariable extension of the Smith predictor, but it only applies to systems with a single delay. The first truly multivariable extension of the Smith predictor was that of Ogunnaike and Ray (1979, 1983). The compensator described in the present paper is a further extension of their structure and was first presented recently by Jerome and Ray (1984). Other important work includes extension of the Smith predictor to a restricted class of nonlinear systems by Herget and Frazier (1980); the inferential model control structure of Brosilow (1979); and, closely related to this, the model scheme of Frank (1974), the dynamic matrix control of Shell (summarized in Cutler et al., 1983), and the internal model control of Garcia and Morari (1984).

In this paper a new multivariable extension of the Smith predictor is described. It has been found that the extension of the properties of the Smith predictor to the multivariable case is not unique. Different compensator designs will result depending on which property of the Smith predictor one wishes to extend to the multivariable case. In order to define the extension used, three important properties of the Smith predictor are described in the next section.

The Smith Predictor

The Smith predictor (Smith, 1957, 1959) is the classical time delay compensator developed for single-input, single-output (SISO) systems. A comparison of the Smith predictor with a conventional process control structure is shown in Figure 1. The additional block in Figure 1 is the delay compensator or predictor block. The output of this block is equal to the difference in the response of the process model with the time delay removed and the response of the model with the time delay retained. Here we denote by $c(s)$ the conventional controller (e.g., PID) where $g^*(s)e^{-\theta s}$ is the process and $\tilde{g}^*(s)e^{-\tilde{\theta}s}$ is the process model. The closed-loop transfer function of the Smith predictor structure of Figure 1 is

$$y = \frac{cg^*e^{-\theta s}}{1 + c\tilde{g}^* + c(g^*e^{-\theta s} - \tilde{g}^*e^{-\tilde{\theta}s})}(y_{set} - d) + d \qquad (1)$$

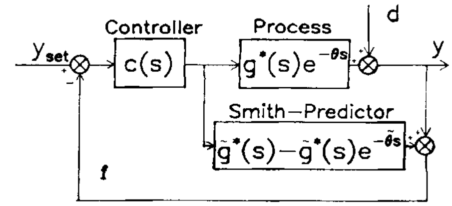

Figure 1. Smith predictor structure.

It is instructive to rearrange Figure 1 into Figure 2. In this case the closed-loop transfer function can be rearranged to

$$y = \frac{\left(\frac{c}{1 + c\tilde{g}^*}\right)g^*e^{-\theta s}}{1 + \left(\frac{c}{1 + c\tilde{g}^*}\right)(g^*e^{-\theta s} - \tilde{g}^*e^{-\tilde{\theta}s})}(y_{set} - d) + d \qquad (2)$$

where the lefthand side controller block becomes

$$u(s) = \frac{c(s)}{1 + c(s)\tilde{g}^*(s)}[y_{set}(s) - d(s)] \qquad (3)$$

and if the controller gain $|c(s)|$ becomes large, then this controller block becomes

$$\lim_{|c|\to\infty} \frac{c(s)}{1 + c(s)\tilde{g}^*(s)^*} = \tilde{g}^*(s)^{-1} \qquad (4)$$

Thus in the limit of infinitely large controller gains, the Smith predictor structure of Figure 2 is similar to other predictive controllers such as the model-based control structure of Brosilow (1979) or the internal model control (IMC) structure of Garcia and Morari (1982) for SISO systems. As shown by Garcia and Morari, any feedback control structure (even PID) can be made to approach their IMC structure at infintely large controller gain. Thus we shall use their idealized controller performance as a measure of best possible performance for the Smith predictor.

When the dynamic model is perfect, (i.e., $g = \tilde{g}$), the closed-loop transfer function for Figures 1–2 becomes:

$$y(s) = \frac{c(s)g^*(s)e^{-\theta s}}{1 + c(s)g^*(s)}[y_{set}(s) - d(s)] + d(s) \qquad (5)$$

So the closed-loop characteristic equation is:

$$1 + c(s)g^*(s) = 0 \qquad (6)$$

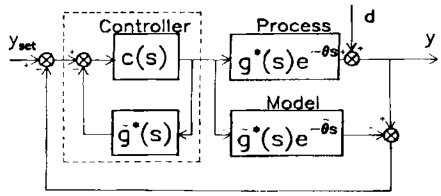

Figure 2. Rearrangement of Smith predictor structure.

which contains no time delays. Hence, with a perfect process model we have:

Property one

The Smith predictor eliminates the time delay from the closed-loop characteristic equation.

This property is often noted, and the success of the Smith predictor is frequently attributed to this property.

Consider the response of the feedback signal, f, in Figure 1–2 to a set point change:

$$f(s) = \frac{c(s)g^*(s)}{1 + c(s)g^*(s)} y_{set}(s) \tag{7}$$

Comparing this with Eq. 5 reveals that in the time domain:

$$f(t) = y(t + \theta) \tag{8}$$

This means that $f(t)$ is predicting the behavior of $y(t)$ by θ time units. Hence, the ideal Smith predictor causes the feedback signal $f(t)$ returned to the controller to be a forecast one time delay into the future of the influence of the controller action on the plant output. This is:

Property two

For set point changes, the Smith predictor provides the controller with an immediate prediction of the effects of its control action on the system output forecast θ time units into the future, $f(t) = y(t + \theta)$.

It should be noted that for disturbances as shown in Figure 1, there is a feedforward as well as a feedback contribution to the signal, f, received by the controller. In this case Eq. 7 becomes

$$f(t) = y(t + \theta) + d(t) - d(t + \theta) \tag{9}$$

so that for rapidly changing disturbances, $d(t)$, this prediction property is lost for the SISO Smith predictor. However, for step or slowly varying disturbances, $d(t) - d(t + \theta) \approx 0$, and the predictive property holds. The problems of designing SISO Smith predictors for disturbance rejection is discussed more fully in Holt and Morari (1985b).

The third observation is that the Smith predictor structure makes an implicit division of the process dynamic model into two parts or factors. The first is the time delay, $e^{-\theta s}$, and the second is all remaining dynamics, $g^*(s)$. These two factors are then treated in fundamentally different ways. To better understand this, consider the broken-line box in Figure 2. The transfer function for this box is given by Eq. 3 or as $|c| \rightarrow \infty$ by Eq. 4, so that

$$u(s) \simeq [g^*(s)]^{-1} (y_{set} - d) \tag{10}$$

and for set point changes, Eq. 5 becomes

$$\begin{aligned} y(s) &= [g^*(s)]^{-1} g^*(s) e^{-\theta s} y_{set}(s) \\ &= e^{\theta s} y_{set}(s) \end{aligned} \tag{11}$$

Of course it may not be possible to make $|c(s)|$ large due to the presence of high-order dynamics, right half-plane zeroes, or constraints on the input. Hence, $|c| \rightarrow \infty$ represents the ideal controller (possibly unrealizable) beyond which no improvement in performance is possible. This ideal controller has been discussed in the context of internal model control (Garcia and Morari, 1982) and provides a useful bound on control system performance. The important point to recognize is that the Smith predictor contains an implicit factorization that has taken the original system (which cannot be inverted without introducing a prediction) and split it into two factors, one of which can be inverted without prediction, and the remainder, which contains the noninvertible part of the original system due to the time delay. Thus we are led to:

Property three

The Smith predictor structure implicitly factors the plant into two parts: $g^*(s)e^{-\theta s} = [e^{-\theta s}][g^*(s)]$. The first, $e^{-\theta s}$, is the noninvertible contribution of the time delay. The second is invertible without prediction.

Property three may seem obvious for the single-input, single-output case, but in the multivariable case more subtlety is required. For example, in a multidelay, multivariable system the mere presence of time delays in a matrix of transfer functions does not necessarily mean that the inverse of that matrix will contain predictions. It is the location and magnitude of the various delays that determine this, and thus the extension of property three to the multivariable case is more complicated.

The Generalized Multidelay Compensator

Ogunnaike et al. (1979, 1983) presented a multidelay compensator (MDC) that showed good performance on a number of simulation and experimental test systems. However, using the same fundamental control system structure, a new generalized multidelay compensator (GMDC) has been developed that shows consistently better performance by extending one or more of the three properties of the Smith predictor to multivariable systems. To introduce and illustrate this new multidelay compensator, we first consider the most common practical case of time delay models with a transfer function matrix of the following form:

$$G(s) = \begin{bmatrix} g_{11}^*(s)e^{-\alpha_{11}s} & \cdots & g_{1n}^*(s)e^{-\alpha_{1n}s} \\ \vdots & & \vdots \\ g_{n1}^*(s)e^{-\alpha_{n1}s} & \cdots & g_{nn}^*(s)e^{-\alpha_{nn}s} \end{bmatrix} \tag{12}$$

where

$$g_{ij}^*(s) = \frac{k_{ij} \prod_{p=1}^{m} (h_{ijp}s + 1)}{\prod_{q=1}^{r} (f_{ijq}s + 1)} \tag{13}$$

and

k_{ij} = steady state gain of element of g_{ij}^*
m = number of zeroes in $g_{ij}^*(s)$
r = number of poles in $g_{ij}^*(s)$

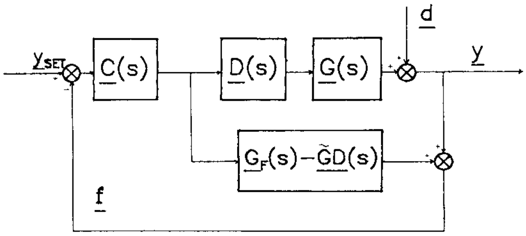

Figure 3. Generalized multidelay compensator (GMDC).

$-1/h_{ijp}$ = pth zero of $g^*_{ij}(s)$
$-1/f_{ijq}$ = qth pole of $g^*_{ij}(s)$
α_{ij} = time delay associated with $g^*_{ij}(s)$

In the latter part of this paper we extend the results to a completely general form of time delay model.

The generalized multidelay compensator (GMDC) is illustrated in Figure 3. This is the same structure originally proposed by Ogunnaike et al. (1979, 1983) but the parameters G_F, D have been generalized. For this structure, we denote the conventional controller by $C(s)$, the process by $G(s)$, the process model by $\tilde{G}(s)$, and the compensator parameters by $G_F(s)$, $D(s)$. Thus the closed-loop transfer function for the GMDC is given by

$$y(s) = GD[I + (I + CG_F)^{-1}C(G - \tilde{G})D]^{-1} \cdot (I + CG_F)^{-1}C \times (y_{set} - d) + d \quad (14)$$

When the model is perfect, this reduces to

$$y(s) = GD(I + CG_F)^{-1} C(y_{set} - d) + d \quad (15)$$

For time delay systems of the form of Eqs. 12 and 13, the $D(s)$ matrix is a diagonal matrix with each element consisting only of a delay:

$$d_{ii}(s) = e^{-\delta_i s} \quad (16)$$

The $G_F(s)$ matrix is identical to the $G(s)$ matrix except for the time delays:

$$G_F(s) = \begin{bmatrix} g^*_{11}(s)e^{-\tau_{11}s} & \cdots & g^*_{1n}(s)e^{-\tau_{1n}s} \\ \vdots & & \vdots \\ g^*_{n1}(s)e^{-\tau_{n1}} & \cdots & g^*_{nn}(s)e^{-\tau_{nn}s} \end{bmatrix} \quad (17)$$

By appropriate choice of delays in the $D(s)$ and $G_F(s)$ matrices, it is possible to extend one or more of the properties of the Smith predictor to the multivariable system described by Eqs. 12 and 13. Simple design rules for the selection of these delays are now presented.

Extension of property one

This is very simple; one need only choose:

$$\delta_i = 0 \quad (i = 1, \ldots, n)$$
$$\tau_{ij} = 0 \quad (i, j = 1, \ldots, n) \quad (18)$$

so that D becomes the identity matrix and $G_F(s)$ is the $G^*(s)$ of Ogunnaike and Ray (1979).

$$G^* = \begin{bmatrix} g^*_{11}g^*_{12} & \cdots & g^*_{1n} \\ \vdots & & \\ g^*_{n1} & \cdots & g^*_{nn} \end{bmatrix} \quad (19)$$

With this choice the GMDC of Figure 3 becomes identical to the multidelay compensator of Ogunnaike and Ray (1979). The characteristic equation with this design is

$$\det (I + CG^*) = 0 \quad (20)$$

where G^*, and thus the characteristic equation, contains no time delays. Hence property one is extended. However, in general, properties two and three are not simultaneously satisfied.

Extension of property two

In the multivariable case property two becomes

$$f_i(t) = y_i(t + \theta_i) \quad i = 1, \ldots, m \quad (21)$$

for a change in y_{set}. This means that each element f_i of the feedback vector is a forecast of the effects of all the controller outputs u on the y_i exactly θ_i time units into the future. The design rule for extension of this property is:

$$\delta_i = 0 \quad (i = 1, 1, \ldots, n) \quad (22)$$

$$\tau_{ij} = \alpha_{ij} - \theta_i \quad (i, j = 1, \ldots, n) \quad (23)$$

where

$$\theta_i = \min_j (\alpha_{ij}) \quad \text{or} \quad G_F = \Theta G(s) \quad (24)$$

where

$$\Theta = \begin{bmatrix} e^{+\theta_1 s} & & 0 \\ & e^{+\theta_2 s} & \\ 0 & & \ddots \; e^{+\theta_n s} \end{bmatrix} \quad (25)$$

This means that again D is the identity matrix and G_F is the same as $\tilde{G}$ with the smallest time delay in each row subtracted from the delays in that row. This procedure insures that G_F is realizable, but accomplishes the prediction of each output as shown in Eq. 21, i.e., the prediction ahead of each y_i by the smallest time delay in the ith row in $G(s)$.

Extension of property three

In the multivariable case, to say property three has been extended means that there is an implicit factorization of $G(s)$ into two parts:

$$G(s) = R(s)G_F(s) \quad (26)$$

which results in both $G_F(s)$, and $G_F^{-1}(s)$, stable and causal (i.e., not involving predictions). The remainder, $R(s)$, contains the noninvertible part of $G(s)$ due to time delays. As in the case of the Smith predictor, G_F may not have a stable inverse because of right half-plane zeroes. However, we indicate below how RHP

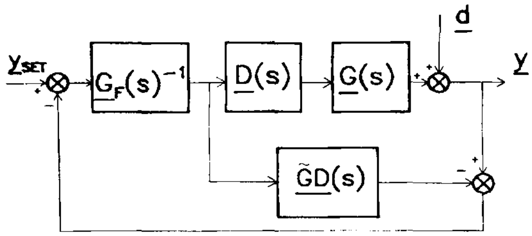

Figure 4. Idealized equivalent of GMDC structure in the limit of large $\|C\|$.

zeroes can be handled as well, and for the present discussion assume no RHP zeroes are present. The design procedure used for extension of property three to the multivariable system, $G(s)$, will depend on the nature of the system.

There are many possible choices of G_F that satisfy property three, and part of the designer's task is to determine the best choice. However, we may be guided in this from the factorization results of Holt and Morari (1984) and Jerome (1985). Neglecting for the moment the fact that, in general, infinite controller gains are not feasible due to controller constraints, RHP zeroes, or high-order dynamics, we shall seek the idealized controller as a performance bound. In the limit as the controller gain becomes infinitely large (i.e., $\|C\| \rightarrow \infty$), the block diagram of Figure 3 takes the form of Figure 4 with the idealized controller

$$G_c = G_F^{-1} \tag{27}$$

so that the idealized response (cf. Eq. 15) becomes for set point changes

$$y(s) = GDG_F^{-1}\, y_{set} \tag{28}$$

and the closed-loop idealized response to set point changes is governed by GDG_F^{-1}. As shown by Garcia and Morari (1982, 1984), any feedback controller can be made to approach the form of an ideal predictive controller such as internal model control (IMC) without filter as the controller gain becomes infinitely large (infinite controller power in each input). Thus we shall use these idealized performance bounds developed by Holt and Morari (1984) as a measure of the relative performance for the GMDC under various designs. Holt and Morari have defined the ideal minimum response times to set point changes when $\|C\| \rightarrow \infty$ and $D \equiv I$. These are as follows:

Theorem 1 (after Holt and Morari)

There is a minimal response time to set point changes, the lower response bound, defined as the smallest delay in the numerators of each row of $G(s)$. That is,

$$L = \begin{bmatrix} e^{-\theta_1 s} & & 0 \\ & \ddots & \\ 0 & & e^{-\theta_n s} \end{bmatrix} \quad \text{where } \theta_i = \min_j (\alpha_{ij}) \tag{29}$$

Similarly, for GD, there is a lower response bound

$$L_d = \begin{bmatrix} e^{-\theta_{d_1} s} & & 0 \\ & \ddots & \\ 0 & & e^{-\theta_{d_n} s} \end{bmatrix} \quad \text{where } \theta_{d_i} = \min_j (\alpha_{ij} + \delta_j) \tag{30}$$

Proof: Follows directly from Holt and Morari (1984).

Theorem 2 (after Holt and Morari):

There is a minimal decoupled response time to set point changes (defined from Eq. 28 when $D = I$) that is the smallest delay in the numerator of each row of GG_F^{-1}. This is defined as the decoupling response bound, R, where

$$R = GG_F^{-1} = \begin{bmatrix} e^{-r_1 s} & & 0 \\ & \ddots & \\ 0 & & e^{-r_n s} \end{bmatrix} \tag{31}$$

and the r_i are chosen as small as possible such that G_F^{-1} does not have predictions. Similarly for GD, when $D \neq I$, the decoupling response bound is

$$R_d = GDG_F^{-1} = \begin{bmatrix} e^{-r_{d_1} s} & & 0 \\ & \ddots & \\ 0 & & e^{-r_{d_n} s} \end{bmatrix} \tag{32}$$

Proof: Follows directly from Holt and Morari (1984).

Definition: We define the rearrangement test for $G(s)$ as whether or not $G(s)$ can be rearranged (through row or column interchanges only) into a matrix with the shortest time delay in each row appearing in the major diagonal.

Theorem 3 (after Holt and Morari)

The lower response bound, L, and the decoupling response bound, R, are equal if and only if $G(s)$ passes the rearrangement test. Similarly, for $GDL_d = R_d$ and only if GD passes the rearrangement test.

Proof: Follows directly from Holt and Morari (1984).

Note that this means that if the rearrangement test is satisfied, then the decoupled response R from Eq. 28 is the best possible response, while if the rearrangement test fails, the best decoupled response R from Eq. 28 may be somewhat inferior to the absolute best response, L.

Because in our designs we find it advantageous to add delays to some inputs, we require a performance bound in this case. This leads to the amazing result that system performance can often be improved or at least not degraded by adding time delays to specific inputs. This is stated in the following theorem.

Theorem 4 (Jerome)

If $D(s)$ is chosen as the diagonal matrix with the smallest delays such that $R^{-1}GD$ has no predictions, or equivalently that GD passes the rearrangement test, then the decoupling response bounds for the original system G and delayed system GD are the same; i.e., $R = R_d (= L_d)$.

Proof: See Jerome (1986).

This striking result means that if we choose D as defined above, we may add time delays (through D) to some inputs and still retain the best decoupled performance of G alone. We shall demonstrate this result in examples below.

Theorem 5 (Jerome)

If $G(s)$ satisfies the rearrangement test, then G_F, (for $D = I$) designed for the prediction property two (eqs. 22–25), will also satisfy property three automatically, and the resulting G_F, G_F^{-1} will have no predictions.

Proof: From Eq. 26

$$G_F = R^{-1}G \tag{26}$$

and from Eq. 25

$$G_F = \oplus G(s) \tag{25}$$

However, when the rearrangement test is satisfied, $\oplus \equiv R^{-1}$, by definition, and hence neither G_F nor G_F^{-1} have predictions.

We shall use these results to develop a design procedure extending property three for each time delay structure that may arise.

Case 1. G(s) *satisfies the rearrangement test*

In this case we may use the results of theorem 5 to simultaneously extend properties two and three by using the design $D(s) = I$, $G_F = \oplus G(s)$. From theorem 3 this will produce the best possible response in the limit as controller gains become infinite. Let us illustrate this case with an example.

Example 1. Wood and Berry Distillation Column

This is the methanol-water distillation column experimentally modeled by Wood and Berry (1973). The transfer function model given for this system is:

$$G(s) = \begin{bmatrix} \frac{12.8e^{-1s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}, \quad G_d(s) = \begin{bmatrix} \frac{3.8e^{-8.1s}}{14.9s+1} \\ \frac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} \tag{33}$$

where $y(s) = G(s)u(s) + G_d d$, and y_1 and y_2 are the overhead and bottoms product mole fraction methanol, while u_1 is the reflux flow rate, u_2 is the reboiler steam flow rate, and d is the feed flow rate. For this system, design of the GMDC according to Eqs. 22–25 leads to:

$$G_F(s) = \begin{bmatrix} \frac{12.8}{16.7s+1} & \frac{-18.9e^{-2s}}{21s+1} \\ \frac{6.6e^{-4s}}{10.9s+1} & \frac{-19.4}{14.4s+1} \end{bmatrix} \tag{34}$$

and $D(s) = I$. Note that since the smallest time delays in Eq. 33 are on the diagonal, property three is also satisfied for this system when design for property two is carried out. In Figure 5 the performance of three different controller designs is compared for a set point change of +0.75 mole fraction percent methanol in the overhead composition y_1. The short-dash curves are conventional PI controllers with no time delay compensation, and with controller settings suggested by Wood and Berry. The long-dash curves represent the performance attained by extension of property one using the MDC of Ogunnaike and Ray (1979) where C is a diagonal matrix of PI controllers. Finally, the solid curves in Figure 5 illustrate the performance attained with the new GMDC employing the extension of properties two and three and where $C(s)$ is a PI controller. Observe the almost total elimination of the upset in y_2 and the superior response in y_1. The controller parameters used in each case are tabulated in Table 1 along with the manipulated variable constraints of Wood and Berry used in all simulations. With the removal of input constraints, response similar to that reported by Garcia and Morari (1984) for the IMC control structure applied to the system of example 1 can be obtained.

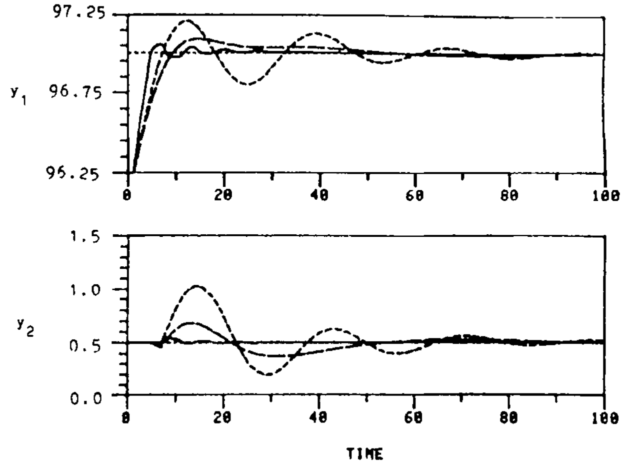

Figure 5a. Response of example 1 system to a set point change of +0.75 mole fraction percent methanol in overhead composition.
...... set points; ---- PI control only; _ _ _ PI with addition of MDC of Ogunnaike and Ray; ____ PI with addition of new GMDC. Tuning parameters given in Table 1

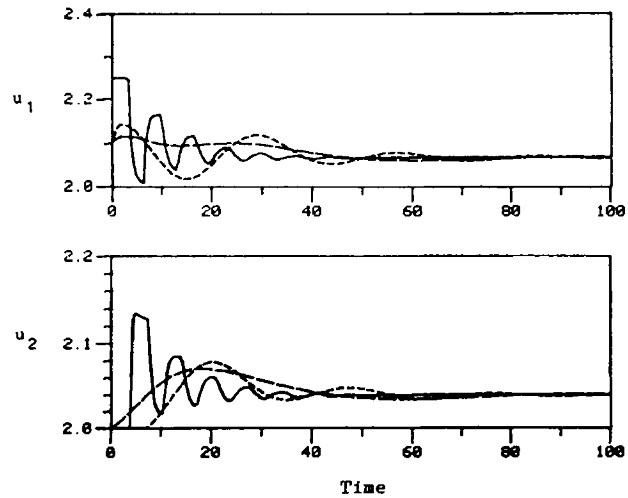

Figure 5b. Controller actions for responses shown in Fig. 5a; legend as in Fig. 5a.

Although we see from Eq. 15 that the introduction of disturbances can disrupt the prediction properties of the GMDC, the ability of the controller to regulate in the face of disturbances can be quite good. As an illustration, Figure 6 compares the performance of the GMDC with the MDC and PI controllers for a pulse in feed rate of +0.34 lbm/min which endures for 20 min and then is removed. Note that the GMDC is clearly superior for disturbance rejections.

Case 2. G(s) *Fails the Rearrangement Test*

When $G(s)$ cannot be rearranged to have the smallest time delay on the diagonal, satisfying property three becomes more complicated. For these cases the designer is faced with a choice. One may design $D(s)$ and $G_F(s)$ to extend both properties two and three for the apparent process GD. This is often adequate.

Table 1. Tuning Parameters and Manipulated Variable Constraints for Examples

Example No.	Controller	Loop #1 K_{c_1}	Loop #1 τ_{I_1}	Loop #2 K_{c_2}	Loop #2 τ_{I_2}	Constraints	Results
1	PI	0.2	4.44	−0.04	2.66	$1.65 \le u_1 \le 2.25$ lbm/min	Figs. 5, 6
	PI + MDC	0.2	4.44	−0.04	2.66	$1.70 \le u_2 \le 2.30$ lbm/min	
	PI + GMDC	1.8	15	−2.8	8		
2	PI	2.4	6.3	4	8.5	$0.068 \le u_1 \le 0.245$ gpm	Fig. 7
	PI + GMDC (Prop. 2)	3.4	6.3	5	8.5	$15.6 \le u_2 \le 34.0$ psig	
	PI + GMDC (Prop. 2 & 3)	20	40	20	15		
3	PI + GMDC (Prop. 2 & 3)	20	40	20	15	$0.068 \le u_1 \le 0.245$ gpm $15.6 \le u_2 \le 34.0$ psig	Fig. 8
	PI + GMDC (Prop. 3)	20	40	20	15		
5	PI	0.2	5.7	0.3	5.0	None	Fig. 9
	PI + GMDC (Prop. 2)	0.75	6.7	2.0	10.0		
	PI + GMDC (Prop. 2 & 3)	14.0	25.0	4.0	6.7		
		K_c	τ_I	K_c	τ_I		
7	PI only	0.6	3.2	0.7	4.0	None	Fig. 10
	PI + GMDC (Prop. 2 & 3)	0.8	4.3	3.8	5.0		
	PI + GMDC (Prop. 2 & 3 & augmentation)	2.4	4.0	4.0	3.3		

The PI controller was implemented in velocity form. SI conversions: kg = lbm × 0.454; L = gal × 3.79; kPa = psig × 6.89.

Alternatively, in special situations one may find it advantageous to satisfy property three and sacrifice property two $f_i(t) = y_i(t + \theta_i)$, for one or more outputs, y_i, and thereby obtain faster responses in the more important outputs. Let us consider each of these cases in more detail.

Design of $D(s)$ to Extend Properties Two and Three for GD. If the designer wishes to guarantee both properties two and three, the basic design procedure for the GMDC is straightforward. First one selects the delays, δ_i, in $D(s)$ so that the apparent system, $G(s)D(s)$, will pass the rearrangement test. The design procedure for accomplishing this (given in the appendix) assures that the diagonal delay matrix $D(s)$ is chosen such that the delays δ_i are the smallest necessary to insure that $G_F = R^{-1}GD$ has no predictions. Implementation of $D(s)$ means delaying some of the process inputs by δ_i time units in order that G_F^{-1} have no predictive elements. Then the delays in $G_F(s)$ are selected according to the following relations:

$$\tau_{ij} = \beta_{ij} - \theta_i \quad (i, j = 1, \ldots, n) \tag{35}$$

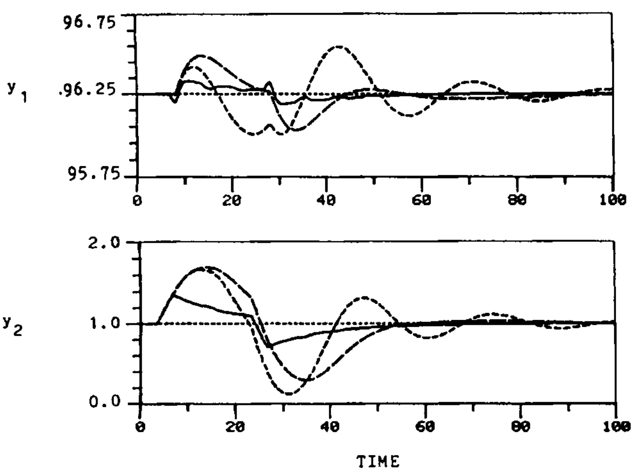

Figure 6a. Response of example 1 system to a 20 min upset in feed flow rate of +0.34.

...... set points; ---- PI control only; — — — PI with addition of MDC of Ogunnaike and Ray; ——— PI with addition of new GMDC. Tuning parameters given in Table 1.

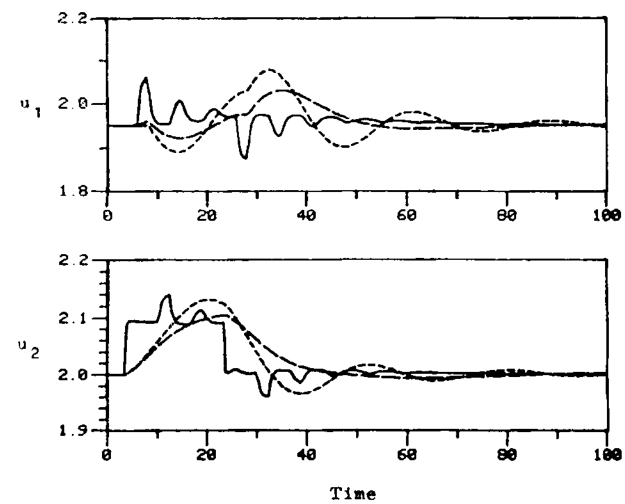

Figure 6b. Controller actions for responses shown in Fig. 6a; legend as in Fig. 6a.

$$\theta_i = \min_j (\beta_{ij}) \tag{36}$$

or

$$G_F = \Theta GD = R^{-1}GD \tag{37}$$

where Θ is defined by Eqs. 25 and 36. Here β_{ij} is the delay associated with the ijth element of the $G(s)D(s)$ matrix. From theorems 4 and 5, this design assures that $G_F(s)^{-1}$ has no predictive elements and guarantees extension of property two also. From theorem 4, the decoupled response from this design, R_d, is equal to the minimal decoupled response of the original system, R. This design procedure has been incorporated into a computer-aided control system design program that automatically designs $G_F(s)$ and $D(s)$ to extend properties two and three for any process $G(s)$. Let us illustrate this case with an example.

Example 2. Ethanol-Water Distillation Column of Ogunnaike

Ogunnaike (1981) has modeled a pilot scale ethanol and water distillation column. His model is as shown below:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.0049e^{-1s}}{9.06s+1} \\ \dfrac{-34.7e^{-9.2s}}{8.15s+1} & \dfrac{0.87(11.61s+1)e^{-1s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \tag{38}$$

Here y_1 is the mole fraction ethanol in the overhead product, y_2 is the temperature on the lowest plate, u_1 is the overhead product flow rate, and u_2 is the steam pressure to the reboiler. This model is completely described in Ogunnaike et al. (1983). Note that it is impossible to choose a loop pairing that puts the smallest delay in each row on the diagonal. This means that direct application of Eqs. 22–25 to extend property two does not simultaneously extend property three. Instead it yields $D(s) = I$ and

$$G_F(s) = \begin{bmatrix} g_{11}^*(s)e^{-1.6s} & g_{12}^*(s) \\ g_{21}^*(s)e^{-8.2s} & g_{22}^*(s) \end{bmatrix} \tag{39}$$

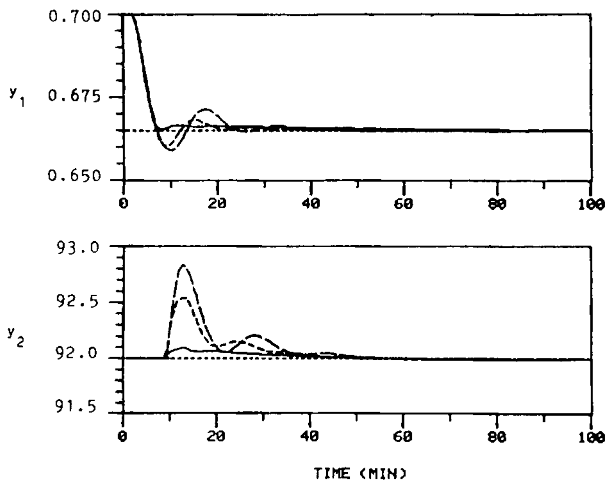

Figure 7a. Response of example 2 system to a set point change of −0.035 mole fraction ethanol in overhead composition.

...... set points; _ _ _ PI control only; ---- PI with GMDC designed for property 2 only; ____ PI with GMDC designed for properties 2 and 3. Tuning parameters given in Table 1.

and the inverse of this $G_F(s)$ does have predictions. Nevertheless, this compensator based only on property two can be applied and does improve controller performance somewhat. However as the controller gain is increased, the closed-loop system approaches G_F^{-1}, which has predictive elements, and the system becomes unstable. In Figure 7 the performance for a change in set point of −0.035 mole fraction percent ethanol for the overhead composition y_1 is compared for three different controller designs. The long-dash curves represent conventional PI loops with no time delay compensation; they are included to provide a basis for comparison. The short-dash curves show the performance obtained with the addition of GMDC designed to extend property two only. The solid lines in Figure 7 show the performance attained when the GMDC is designed to extend both properties two and three. Following the procedure described in the appendix, the $G_F(s)$ and $D(s)$ required to allow the smallest time delay of G_FD to be on the diagonal is

$$G_F(s) = \begin{bmatrix} g_{11}^*(s) & g_{12}^*(s) \\ g_{21}^*(s)e^{-6.6s} & g_{22}^*(s) \end{bmatrix} \tag{40}$$

and

$$D(s) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-1.6s} \end{bmatrix} \tag{41}$$

Note in Figure 7 the excellent performance of this latter case over the other two controllers. It should be noted that when property three is extended in examples 1 and 2, and if the constraints on the controller action were removed, it would be possible to make $\|C(s)\|$ very large and to approach $[G_F(s)]^{-1}$ very closely, resulting in perfect dynamically decoupled response.

To illustrate the simplicity of implementation of the GMDC to this example, we shall explicitly describe the equation that must be solved in real time. In practice two blocks must be added to the traditional control scheme. The $D(s)$ block is implemented by delaying the appropriate inputs, while the compensator $(G_F - GD)$ is implemented by finding a realization.

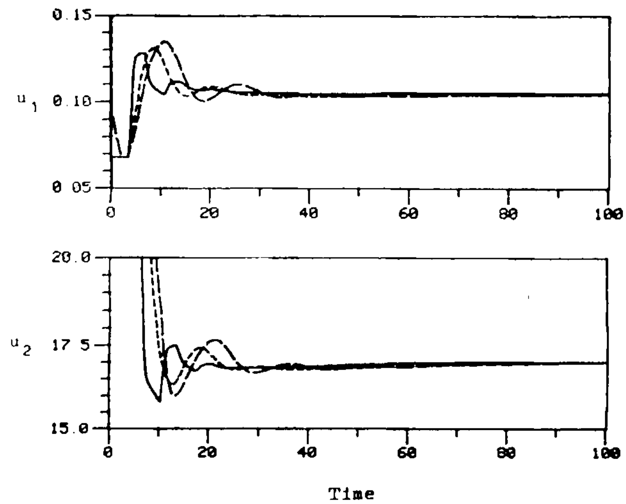

Figure 7b. Controller actions for responses shown in Fig. 7a; legend as in Fig. 7a.

For example 2, the design Eqs. 40 and 41 require that u_2 be delayed by 1.6 min always, and the compensator block $G_F - GD$ in Figure 3 is the set of equations:

$$\begin{aligned}
\dot{x}_1 &= -0.14925\, x_1 + 0.098507\,[u_1(t) - u_1(t-2.6)] \\
\dot{x}_2 &= -0.11038\, x_2 + 0.00054084\,[u_2(t) - u_2(t-2.6)] \\
\dot{x}_3 &= -0.12270\, x_3 - 4.2577\,[u_1(t-6.6) - u_2(t-9.2)] \\
\dot{x}_4 &= -0.053191\, x_4 + 0.022316\,[u_2(t) - u_2(t-2.6)] \\
\dot{x}_5 &= -0.25707\, x_5 + 0.11580\,[u_2(t) - u_2(t-2.6)] \\
z_1 &= x_1 + x_2 \\
z_2 &= x_3 + x_4 + x_5
\end{aligned} \tag{42}$$

where z is the output of the compensator. Note that these equations are uncoupled, so that only an integration is necessary in each case.

Design to Emphasize Performance of Some Outputs Over Others. We now consider situations where the system fails the rearrangement test and the $D(s)$ matrix required for simultaneous extension of properties two and three (complete dynamic decoupling) has delays that are considered too long. In these cases (as shown in theorems 1–3) in the limit of infinite controller gain, the fastest possible response for some outputs is not achieved with dynamic decoupling, but rather through some type of interactive controller. Thus it is possible to improve the response of some outputs at the expense of others. In terms of compensator design this means that property three is retained but property two is sacrificed for some of the outputs in order to improve the response of the most important outputs. This special case can lead to many possible designs; however, we recommend here a particular design that retains property two for the most favored outputs. The procedure is as follows:

1. In order to improve the response of outputs y^+ at the expense of outputs y^- where

$$y = \begin{bmatrix} y^+ \\ --- \\ y^- \end{bmatrix} \begin{matrix} p \text{ vector} \\ \\ n-p \text{ vector} \end{matrix} \tag{43}$$

we should rearrange $G(s)$ such that

$$\begin{bmatrix} y^+(s) \\ ---- \\ y^-(s) \end{bmatrix} = \begin{bmatrix} p \times n \\ G^+(s) \\ -------- \\ (n-p) \times n \\ G^-(s) \end{bmatrix} u(s) \tag{44}$$

by interchange of rows as necessary.

2. Next a $D(s)$ matrix is selected such that for:

$$GD = \begin{bmatrix} G^+D(s) \\ ------ \\ G^-(s)D(s) \end{bmatrix} \tag{45}$$

the top part, G^+D, satisfies the rearrangement test. This $D(s)$ may be determined by a search over the elements of D, δ_i to find the smallest delays necessary, or equivalently by application of the procedure described in the appendix for extension of properties two and three to the matrix:

$$\begin{bmatrix} G^+(s) \\ ---- \\ G^{-*}(s) \end{bmatrix} \tag{46}$$

where $G^{-*}(s)$ is equal to $G^-(s)$ with all delays removed. Note that this will in general require smaller delays in D than required to have the complete matrix $G(s)$ satisfy the rearrangement test.

3. Then we design G_F as follows

$$G_F = \begin{bmatrix} G_F^+ \\ ---- \\ G_F^- \end{bmatrix} \tag{47}$$

where

$$G_F^+ = \Theta^+ G^+ D \tag{48}$$

and Θ^+ is a matrix composed of delays, $e^{+\theta_i s}$ along the diagonal, where θ_i is equal in magnitude to the shortest delay in the ith row of $G^+(s)D$.

Having designed for y^+ we must now choose G_F^- such that G_F given by Eq. 47 and its inverse are both causal and stable and yet sacrifice as little of property two as possible for the y^- outputs. This can be accomplished by using G_F in the form of Eq. 17 and selecting the τ_{ij} of G_F (with the τ_{ij}^+ of G_F^+ fixed above) such that the τ_{ij} are as close as possible to the α_{ij} of $G(s)$ and yet satisfy property three. Because we wish the response for y^- to be as rapid as possible and G_F^- to produce an approximation to a predictor for y^-, we wish to choose the τ_{ij} of G_F^- to be as small as possible and the differences $\tau_{ij} - \tau_{ik}$ within a given row be as close as possible to the differences $\alpha_{ij} - \alpha_{ik}$ in the original system $G^-(s)D$. Obviously, if $G(s)D(s)$ were to satisfy the rearrangement test, then this design for $G_F(s)^-$ would exactly satisfy properties two and three simultaneously.

We begin with $G_F^- = G^-$ and start by reducing the magnitude of every delay in row i of G_F^- by the magnitude of the shortest delay in row i of G_F^- for each row in G_F^-. If $[G_F]^{-1}$ now does not predict we stop; otherwise we continue by picking the least important element of y^- and doing the following to the corresponding row (call it row j) in $G_F^-(s)$: Shorten every delay in row j by an amount equal to the magnitude of the smallest nonzero delay in row j. (If a delay is already zero, it remains zero.) Note that property two has now been lost for this row and this element of y^-. Check $[G_F]^{-1}$ again for prediction. Continue this process on this row until $[G_F]^{-1}$ has no prediction (in which case we are finished) or until all delays are gone from row j. Next move to the row of G_F^- associated with the next least important element of y^- and do the same thing. Eventually a $[G_F]^{-1}$ without predictions will be obtained since the limit of this process results in $G_F^- = G^{-*}$. (Note: If different loop pairings are desired, identical row interchanges may be performed on the $G(s)$, $G_F(s)$ and $D(s)$ matrices.)

This procedure has the advantage that as little of property two as possible is lost, and then only for the least important outputs. The procedure is illustrated in the following two examples.

Example 3 (A variation on the distillation column in example 2)

Suppose

$$G(s) = \begin{bmatrix} g_{11}^*(s)e^{-6s} & g_{12}^*(s)e^{-1s} \\ g_{21}^*(s)e^{-9.2s} & g_{22}^*(s)e^{-1s} \end{bmatrix} \tag{49}$$

where the dynamic parts $g_{ij}^*(s)$ are identical to the system of example 2. Note that this system fails the rearrangement test. The compensator design for simultaneous extension of properties two and three is:

$$D(s) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-5s} \end{bmatrix} \quad G_F(s) = \begin{bmatrix} g_{11}^*(s) & g_{12}^*(s) \\ g_{21}^*(s)e^{-3.2s} & g_{22}^*(s) \end{bmatrix} \tag{50}$$

With this design, as $\|C(s)\|$ becomes large, the closed-loop transfer function approaches the decoupling performance limit of:

$$R = \begin{bmatrix} e^{-6s} & 0 \\ 0 & e^{-6s} \end{bmatrix} \tag{51}$$

This is illustrated by the short-dash curves of Figure 8 for a change in the set point of y_2. Perfect decoupling could have been achieved if the constraints on the controller actions had been removed.

Now assume that y_2 is more important than y_1 and so we are willing to tolerate interactions on y_1 if we can improve the response of y_2. Choosing our partition so that $y^+ = y_2$ and carrying out the three-step procedure described above results in the following compensator design.

Let us interchange rows in $G(s)$ because $y^+ = y_2$, $y^- = y_1$, then

$$G = \begin{bmatrix} G^+ \\ \hline G^- \end{bmatrix} = \begin{bmatrix} g_{21}^*e^{-9.2s} & g_{22}^*e^{-s} \\ \hline g_{11}^*e^{-6s} & g_{12}^*e^{-s} \end{bmatrix} \tag{52}$$

and our design is

$$D(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad G_F^+ = [g_{21}^*e^{-8.2s} \quad g_{22}^*] \tag{53}$$

where the parameters $\tau_{11}\tau_{12}$ in G_F^- must be chosen so that

$$G_F = \begin{bmatrix} g_{21}^*e^{-8.2s} & g_{22}^* \\ \hline g_{11}^*e^{-\tau_{11}s} & g_{12}^*e^{-\tau_{12}s} \end{bmatrix} \tag{54}$$

and

$$(G_F)^{-1} = \frac{\begin{bmatrix} g_{12}^*e^{-\tau_{12}s} & -g_{22}^* \\ -g_{11}^*e^{-\tau_{11}s} & g_{21}^*e^{-8.2s} \end{bmatrix}}{\{g_{12}^*g_{21}^* \exp[-(8.2s + \tau_{12})s] - g_{11}^*g_{22}^*e^{-\tau_{11}s}\}} \tag{55}$$

are both causal and stable. According to the design procedure above, the best choice of the τ_{ij} of G_F^- is that one which makes the difference in time delays in a given row as close as possible to the differences in the original rows of $G(s)$ and the absolute values of τ_{ij} as small as possible. Thus for this example the original system G^- has

$$\alpha_{11} = 6, \quad \alpha_{12} = 1, \quad \alpha_{11} - \alpha_{12} = 5$$

However any $\tau_{11} > 0$ causes G_F^{-1} to be noncausal. Hence the starting form $G_F^- = [g_{11}^*e^{-5s} g_{12}^*]$ has $[G_F]^{-1}$ noncausal. Thus G_F^- design requires a further iteration, resulting in the final $G_F^- = [g_{11}^* \; g_{12}^*]$. Rearranging back to the original loop pairing gives:

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad G_F = \begin{bmatrix} g_{11}^*(s) & g_{12}^*(s) \\ g_{21}^*(s)e^{-8.2s} & g_{22}^*(s) \end{bmatrix} \tag{56}$$

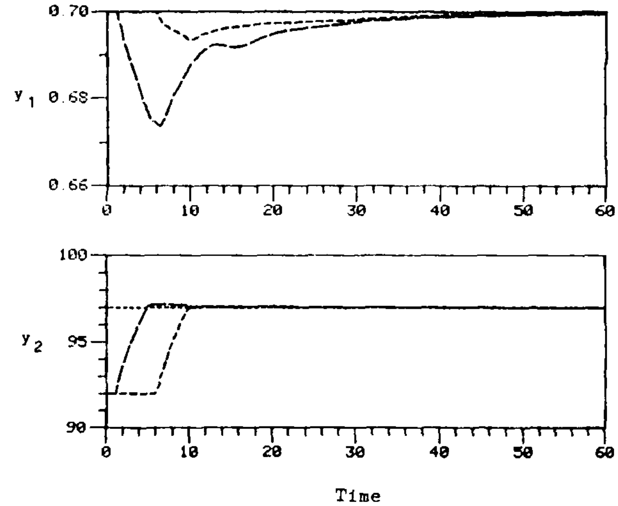

Figure 8a. Response of example 3 system to a step of +5.0 in the set point of y_2.

...... set points; ---- PI with GMDC extending properties 2 and 3; — — — PI with GMDC extending property 3 only (favoring y_2). Tuning parameters given in Table 1.

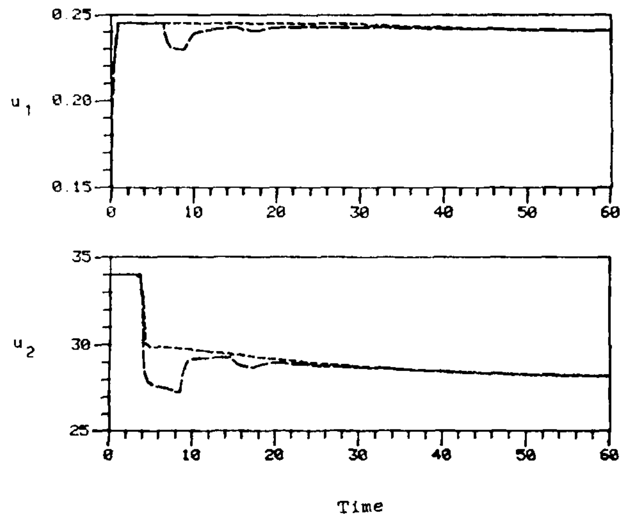

Figure 8b. Controller actions for responses shown in Fig. 8a; legend as in Fig. 8a.

Note that with this design property two is lost for y_1 and hence only property three is extended. The performance of this alternate design is shown in Figure 8 by the long-dash curves. y_2 now responds after a delay of only 1 min (the lower response limit) as opposed to the 6 min delay observed before. The price paid is the increased interaction now present in y_1 that was absent before.

Example 4

Ogunnaike et al. (1983) present this model of a pilot-scale distillation column. (Example 2 is a simplification of this system.)

$$G(s) = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.5s}}{8.6s+1} & \dfrac{-0.005e^{-1s}}{9.1s+1} \\ \dfrac{1.11e^{-6.5s}}{3.2s+1} & \dfrac{-2.36e^{-3s}}{5.0s+1} & \dfrac{-0.012e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.7e^{-9.2s}}{8.1s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.6s+1)e^{-1s}}{(3.9s+1)(18.8s+1)} \end{bmatrix} \tag{57}$$

Note that $G(s)$ fails the rearrangement test. Thus the compensator design that extends properties two and three resulting from the design procedure above requires

$$D(s) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-1.8s} \end{bmatrix} \tag{58}$$

and

$$G_F(s) = \begin{bmatrix} g_{11}^*(s) & g_{12}^*(s)e^{-0.9s} & g_{13}^*(s)e^{-0.2s} \\ g_{21}^*(s)e^{-3.5s} & g_{22}^*(s) & g_{23}^*(s) \\ g_{31}^*e^{-6.4s} & g_{32}^*(s)e^{-6.6s} & g_{33}^*(s) \end{bmatrix} \tag{59}$$

This design has as its limiting performance for large $\|C(s)\|$, the dynamically decoupled closed-loop response of

$$R(s) = \begin{bmatrix} e^{-2.6s} & 0 & 0 \\ 0 & e^{-3.0s} & 0 \\ 0 & 0 & e^{-2.8s} \end{bmatrix} \tag{60}$$

Suppose now that the bottom tray temperature y_3 is considered unimportant. Let us determine what improvement can be made in the responses of y_1 and y_2 if interactions are allowed on y_3. This corresponds to the following partition:

$$\begin{bmatrix} y^+ \\ \hline y^- \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \hline y_3 \end{bmatrix} = \begin{bmatrix} G^+(s) \\ \hline G^-(s) \end{bmatrix} u(s)$$

$$= \begin{bmatrix} g_{11}^*e^{-2.6s} & g_{12}^*e^{-3.5s} & g_{13}^*e^{-1.0s} \\ g_{21}^*e^{-6.5s} & g_{22}^*e^{-3.0s} & g_{23}^*e^{-1.2s} \\ \hline g_{31}^*e^{-9.2s} & g_{32}^*e^{-9.4s} & g_{33}^*e^{-1.0s} \end{bmatrix} u(s) \tag{61}$$

Carrying out the above design procedure for extension of property three with this partition yields the following design:

$$D(s) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-1.6s} \end{bmatrix} \tag{62}$$

and

$$G_F(s) = \begin{bmatrix} g_{11}^*(s) & g_{12}^*(s)e^{-0.9s} & g_{13}^*(s) \\ g_{21}^*(s)e^{-3.7s} & g_{22}^*(s)e^{-0.2s} & g_{23}^*(s) \\ g_{31}^*(s) & g_{32}^*(s) & g_{33}^*(s) \end{bmatrix} \tag{63}$$

This design has as its limiting performance for large $\|C(s)\|$, the closed-loop response of:

$$R(s) = \begin{bmatrix} e^{-2.6s} & 0 & 0 \\ 0 & e^{-2.8s} & 0 \\ a_1(s) & a_2(s) & b(s) \end{bmatrix} \tag{64}$$

where $a_1(s)$ and $a_2(s)$ are complicated dynamic interactions and $b(s)$ is a complicated transfer function. Comparison with Eq. 60 shows that for this design, allowing interactions on y_3 only improved the limiting response for y_2 by 0.2 time units. Thus the decoupled response extending properties two and three is almost as good as extending property three alone for this example.

Suppose now that instead of y_3 being least important, it is most important and y_1 and y_2 are of less importance. We now see what improvement is possible for y_3 when interactions are allowed on y_1 and y_2. This corresponds to the following partition:

$$y = \begin{bmatrix} y^- \\ \hline y^+ \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \hline y_3 \end{bmatrix} \tag{65}$$

Carrying out the design procedure for extension of property three with this partition and the assumption that y_2 is of least importance yields the following design:

$$D(s) = I$$

and

$$G_F(s) = \begin{bmatrix} g_{11}^*(s) & g_{12}^*e^{-0.9s} & g_{13}^*(s) \\ g_{21}^*(s) & g_{22}^*(s) & g_{23}^*(s) \\ g_{31}^*(s)e^{-8.2s} & g_{32}^*(s)e^{-8.4s} & g_{33}^*(s) \end{bmatrix} \tag{66}$$

The limiting closed-loop response for this design is:

$$R(s) = \begin{bmatrix} b_1(s) & a_1(s) & a_2(s) \\ a_3(s) & b_2(s) & a_4(s) \\ 0 & 0 & e^{-1s} \end{bmatrix} \tag{67}$$

where $a_i(s)$ are dynamic interactions and $b_i(s)$ are complicated transfer functions. Note the improved response of y_3 over the diagonal decoupling limit of Eq. 60.

Compensator Design for More General Time Delay Models

In order for the formulation to this point be clear, we have limited ourselves to models of the form of Eqs. 12 and 13. However, the results discussed so far are easily extended to a more general class of models that allow time delays to appear in any form. Thus let us consider linear transfer function models in the general form

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ \vdots & & & \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn(s)} \end{bmatrix} \tag{68}$$

where each element can have the form:

$$g_{ij}(s) = \frac{\sum_{k=1}^{M_{ij}} p(s)_{ijk} e^{-(\alpha_{ijk})s}}{q(s)_{ij1} + \sum_{\ell=2}^{L_{ij}} q(s)_{ij\ell} e^{-(\beta_{ij\ell})s}} = \frac{P_{ij}(s)}{Q_{ij}(s)} \tag{69}$$

here:

$p(s)_{ijk}$ = a polynomial
$q(s)_{ij\ell}$ = a polynomial
M_{ij} = the number of terms in the numerator of g_{ij}
L_{ij} = the number of terms in the denominator of g_{ij}
α_{ijk} = the delay of the kth term in the numerator
$\beta_{ij\ell}$ = the delay of the ℓth term in the denominator

Note that in this formulation, $\beta_{ij1} = 0$ or equivalently, $q_{ij1}(s)$ has no delay associated with it. This is not a limitation since one may always divide numerator and denominator by the smallest delay $\beta_{ij\ell}$ to obtain this form. Note that if the process in this form has one or more of the $\alpha_{ijk} < 0$, then this element $g_{ij}(s)$ contains a prediction and is not a physically acceptable model.

We shall now show that extending the three properties of the Smith predictor to this generalized model is straightforward.

1. *Property One.* To remove the delays from the closed-loop characteristic equation, it is sufficient to remove all time delays from $G(s)$ and use $G_F = G^*(s)$ (i.e., all $\alpha_{ijk} \equiv 0, \beta_{ij\ell} \equiv 0$) as in the previous case. This trivially extends property three as well.

2. *Property Two.* In order to choose G_F, D so that the feedback signal, $f_i(t)$, is a prediction for each output, $y_i(t)$, we must determine the smallest time delay in any of the terms in the numerators of the ith row, i.e.,

$$\theta_i = \min_j \min_k \alpha_{ijk} \tag{70}$$

Then by selecting $D(s) = I$ and

$$G_F(s) = \oplus G D \tag{71}$$

with elements

$$g_{Fij}(s) = \frac{\sum_{k=1}^{M_{ij}} p_{ijk}(s) e^{-(\alpha_{ijk}-\theta_i)s}}{q_{ij1}(s) + \sum_{\ell=2}^{L_{ij}} q_{ij\ell}(s) e^{-\beta_{ij\ell}s}} \tag{72}$$

we again produce the prediction

$$f_i(t) = y_i(t + \theta_i) \quad i = 1, 2, \ldots, n \tag{73}$$

Table 2. Design Procedure for the Generalized Multidelay Compensator

Property 1. All delays eliminated from the closed-loop characteristic equation.
Property 2. Feedback signals are a forecast of the effects of the controller actions on outputs, $f_i(t) = y_i(t + \theta_i)$.
Property 3. G_F is a minimal time delay factorization of $G(s)$ such that both $G_F(s)$ and $G_F^{-1}(s)$ contain no predictions.

Extended Property	Design (for $G(s)$ in form of Eqs. 12, 13)	Design (for $G(s)$ in form of Eqs. 68, 69)
Prop. 1	$D(s) = I$; $G_F(s) = G^*(s)$	$D(s) = I$; $G_F(s) = G^*(s)$
Prop. 2	$D(s) = I$; $G_F(s) = \oplus G(s)$ where $\oplus = \begin{bmatrix} e^{+\theta_1 s} & & 0 \\ & \ddots & \\ 0 & & e_{+\theta_n s} \end{bmatrix}$, $\theta_i = \min_j (\alpha_{ij})$	$D(s) = I$; $G_F(s) = \oplus G(s)$ where $\oplus = \begin{bmatrix} e^{+\theta_1 s} & & 0 \\ & \ddots & \\ 0 & & e_{+\theta_n s} \end{bmatrix}$, $\theta_i = \min_j \min_k (\alpha_{ijk})$
Prop. 3	**Case 1** $G(s)$ Passes the Rearrangement Test The design for extension of property 2 also extends property 3 (both properties 2 and 3 satisfied for G).	$T(s)$ Passes the Rearrangement Test The design for extension of property 2 also extends property 3 (both properties 2 and 3 satisfied for G).
	Case 2: $G(s)$ Fails the Rearrangement Test (a) $D(s) = \text{diag}\,[e^{-\delta_1 s} \cdots e^{-\delta_n s}]$ with δ_i's chosen to eliminate all predictions from $R^{-1}(s)$ $G(s)$ $D(s)$ where $R(s)$ is the decoupling performance limit for $G(s)$; $G_F(s) = R^{-1}(s)\, G(s)\, D(s)$ (both properties 2 and 3 satisfied for GD) (b) Improve performance of some outputs y^+ at expense of others y^- by extending only property 3. $D(s)$ chosen to make G^+D pass rearrangement test. G_F^+ chosen according to Eq. 48 and G_F^- chosen to make G_F and $(G_F)^{-1}$ causal and stable with delays as close to those in G^- as possible. 1, 2, and 3* Let $G_F(s) = G^*$ $D(s) = [G^{-1}(s)\, R(s)]\, G_F(s)$	$T(s)$ Fails the Rearrangement Test (a) $D(s) = \text{diag}\,[e^{-\delta_1 s} \cdots e^{-\delta_n s}]$ with δ_i's chosen to eliminate all predictions from $R^{-1}T(s)$ $D(s)$ where $R(s)$ is the decoupling performance limit for $G(s)$; $G_F(s) = R^{-1}(s)\, G(s)\, D(s)$ (both properties 2 and 3 satisfied for GD) (b) Improve performance of some outputs y^+ at expense of others y^- by extending only property 3. $D(s)$ chosen to make T^+D pass rearrangement test. G_F^+ chosen according to Eq. 48 and G_F^- chosen to make G_F and $(G_F)^{-1}$ causal and stable with delays as close to those in G^- as possible. Let $G_F(s) = G^*$ $D(s) = [G^{-1}(s)\, R(s)]\, G_F(s)$

*D is no longer limited to a diagonal matrix of delays in this case.

for a set point change. Thus $f_i(t)$ is a forecast of the effects of all the controller outputs $\boldsymbol{u}(t)$ or the output y_i exactly θ_i time units into the future.

3. *Property Three*. Here we wish to provide a factorization of $G(s)$

$$G(s) = \boldsymbol{R}(s)\boldsymbol{G}_F(s) \tag{74}$$

where both $\boldsymbol{G}_F(s)$ and $\boldsymbol{G}_F(s)^{-1}$ are causal (no prediction) and stable while $\boldsymbol{R}(s)$ is a remainder containing the part of $\boldsymbol{G}(s)$ that is noninvertible due to time delays. As shown in theorem 6 in the appendix, the predictive nature of $\boldsymbol{G}(s)$ depends on the smallest time delay in the numerators of each row. Thus we may define a $n \times n$ test matrix $\boldsymbol{T}(s)$ whose elements are given by

$$t_{ij} = t_{ij}^* e^{-\tilde{\alpha}_{ij}s} \tag{75}$$

where t_{ij}^* = a parameter chosen such that $t_{ij}^* = 0$ if $g_{ij}(s) \equiv 0$, and to allow no exact cancellation of terms in manipulation of $\boldsymbol{T}(s)$, $\tilde{\alpha}_{ij} = \min_k \alpha_{ijk}$, the smallest delay in the numerator of $g_{ij}(s)$. Then, the time delay properties with respect to prediction are the same for $\boldsymbol{G}(s)$ and $\boldsymbol{T}(s)$.

Thus we have the following important results:

1. If the rows and columns $\boldsymbol{T}(s)$ may be rearranged to place the smallest delay $\tilde{\alpha}_{ij}$ on the diagonal, then the design procedure for property two alone also guarantees property three.

2. If $\boldsymbol{T}(s)$ fails this rearrangement test, then one may choose to design $\boldsymbol{D}(s)$ to cause $\boldsymbol{G}_F\boldsymbol{D}$ to pass the rearrangement test (see the appendix for this procedure). This design provides diagonally decoupled response satisfying both properties two and three, but as with the simple model above, decoupled responses may be suboptimal.

3. Alternatively, if $\boldsymbol{T}(s)$ fails the rearrangement test, then one may wish to sacrifice the predictive property two for some outputs and retain only property three in order to improve the response of some crucial outputs at the expense of less critical ones.

The design procedures are derived in the appendix and summarized in Table 2. Hence, all the results of the simple time delay model extend to the general form.

Let us illustrate the results for the more general model through an example:

Example 5

Consider the following $G(s)$ model, which contains elements of the general form:

$$\boldsymbol{G}(s) = \begin{bmatrix} \dfrac{(s+1)e^{-6s} + (s+2)e^{-7s}}{16s^2 + 8.5s + 1} & \dfrac{0.5e^{-4s}}{25s^2 + 12s + 1} \\ \dfrac{0.5e^{-4s}}{25s^2 + 12s + 1} & \dfrac{(s+2)e^{-3s} + (s+3)e^{-2s}}{(6s+1) + (0.5s+1)e^{-5s}} \end{bmatrix} \tag{76}$$

We give the results of application of the various procedures described above to extend the different properties.

Property One

$$\text{Let } \boldsymbol{D}(s) = \boldsymbol{I} \tag{77}$$

and

$$\boldsymbol{G}_F(s) = \boldsymbol{G}^*(s) = \begin{bmatrix} \dfrac{2s+3}{16s^2 + 8.5s + 1} & \dfrac{0.5}{25s^2 + 12s + 1} \\ \dfrac{0.5}{25s^2 + 12s + 1} & \dfrac{2s+5}{6.5s+2} \end{bmatrix} \tag{78}$$

Property Two

$$\text{Let } \boldsymbol{D}(s) = \boldsymbol{I} \tag{79}$$

Then the test matrix $\boldsymbol{T}(s)$ is

$$\boldsymbol{T}(s) = \begin{bmatrix} t_{11}^* e^{-6s} & t_{12}^* e^{-4s} \\ t_{21}^* e^{-4s} & t_{22}^* e^{-2s} \end{bmatrix} \tag{80}$$

and

$$\boldsymbol{G}_F(s) = \begin{bmatrix} \dfrac{(s+1)e^{-2s} + (s+2)e^{-3s}}{16s^2 + 8.5s + 1} & \dfrac{0.5}{25s^2 + 12s + 1} \\ \dfrac{0.5e^{-2s}}{25s^2 + 12s + 1} & \dfrac{(s+2)e^{-1s} + (s+3)}{(6s+1) + (0.5s+1)e^{-5s}} \end{bmatrix} \tag{81}$$

Here the delays in the numerators of row 1 of $\boldsymbol{G}_F(s)$ are less than those of $\boldsymbol{G}(s)$ by 4.0 time units, since this is the shortest delay in the first row of $\boldsymbol{T}(s)$. Similarly, the delays in the numerators of row 2 are 2.0 time units shorter than in $\boldsymbol{G}(s)$ because the shortest delay in row 2 of $\boldsymbol{T}(s)$ is 2.0.

Properties Two and Three

Note that the test matrix $\boldsymbol{T}(s)$ given in Eq. 80 fails the rearrangement test. This means that the design above for direct extension of property two will not give property three as well. So a nonidentity $\boldsymbol{D}(s)$ matrix will be required for simultaneous extension of these properties. To accomplish this, one may apply the procedure described in the appendix to yield:

$$\boldsymbol{D}(s) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-2s} \end{bmatrix} \tag{82}$$

and

$$\boldsymbol{G}_F(s) = \begin{bmatrix} \dfrac{(s+1) + (s+2)e^{-1s}}{16s^2 + 8.5s + 1} & \dfrac{0.5}{25^2 + 12s + 1} \\ \dfrac{0.5}{25s^2 + 12s + 1} & \dfrac{(s+2)e^{-1s} + (s+3)}{(6s+1) + (0.5s+1)e^{-5s}} \end{bmatrix} \tag{83}$$

Performance of the last two designs is contrasted to that of conventional proportional-integral control for a step in the set point of the second output in Figure 9. Extension of property two alone gives a large improvement over conventional control. Note that extension of both properties two and three virtually eliminates the upset in the first output, but at the expense of an additional delay in the response of y_2 of 2.0 time units [the delay of element (2, 2) in $\boldsymbol{D}(s)$].

More General Design of $D(s)$, $G_F(s)$

It is possible to liberalize the design constraints on $\boldsymbol{D}(s)$, $\boldsymbol{G}_F(s)$ and in so doing produce more complex time delay compensators. For example, to insure that all three properties of the SISO Smith predictor are extended simultaneously, we can choose $\boldsymbol{D}(s)$, $\boldsymbol{G}_F$ such that for each row of $\boldsymbol{GD}$ the time delays in that row are identical. One design that does this is

$$G_F = G^*(s) \tag{84}$$

and $\boldsymbol{D}(s)$ (nondiagonal) of the form

$$D(s) = G^{-1}(s)E(s)G^*(s) \tag{85}$$

Thus $\boldsymbol{GD} = \boldsymbol{E}(s)\boldsymbol{G}^*$ where $\boldsymbol{E}(s)$ is a diagonal matrix of time delays whose elements are

$$E(s) = \text{diag}\,[e^{-r_1 s}, e^{-r_2 s}, \ldots, e^{-r_n s}] \tag{86}$$

where

$$r_j = \max_i \,[\max(0, a_{ij} - b_{ij})] \tag{87}$$

and

a_{ij} = minimum delay in numerator of (ij)th element of $\boldsymbol{G}^{-1}(s)$

b_{ij} = minimum delay in denominator of (ij)th element of $\boldsymbol{G}^{-1}(s)$

Here r_j corresponds to the largest prediction in the jth column of $\boldsymbol{G}^{-1}(s)$.

This design does extend all three properties of the Smith predictor, but also creates a much more complex controller design. This can be illustrated by considering again the Ogunnaike-Ray distillation column example.

Example 6

Recall from example 2 that the model, $\boldsymbol{G}(s)$, for this distillation column is given by Eq. 30 and this fails the time delay rearrangement test. Using the present design to extend properties one, two, and three, one obtains

$$G_F = G^* = \begin{bmatrix} g_{11}^*(s) & g_{12}^*(s) \\ g_{21}^*(s) & g_{22}^*(s) \end{bmatrix} \tag{88}$$

$$G^{-1}(s) = \frac{\begin{bmatrix} g_{22}^* e^{-s} & -g_{12}^* e^{-s} \\ -g_{21}^* e^{-9.2s} & g_{11}^* e^{-2.6s} \end{bmatrix}}{g_{11}^* g_{22}^* e^{-3.6s} - g_{12}^* g_{21}^* e^{-10.2s}} \tag{89}$$

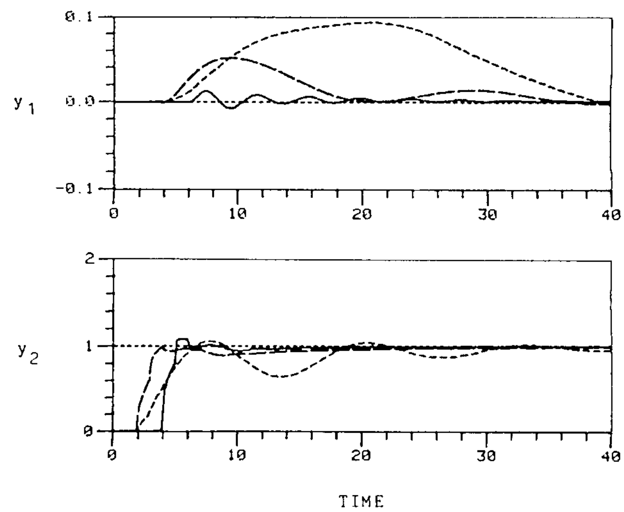

Figure 9. Response of example 5 system to a step of +1.0 in set point of y_2.

...... set points; ---- PI control only; _ _ _ _ PI with addition of GMDC extending property 2 only; ____ PI with addition of GMDC extending properties 2 and 3. Tuning parameters given in Table 1.

so that $\boldsymbol{E}(s)$ becomes

$$E(s) = \begin{bmatrix} e^{-2.6s} & 0 \\ 0 & e^{-2.6s} \end{bmatrix} \tag{90}$$

and $\boldsymbol{D} = \boldsymbol{G}^{-1}\boldsymbol{E}\boldsymbol{G}^*$ becomes

$$D(s) = \frac{\begin{bmatrix} g_{11}^* g_{22}^* e^{-3.6s} - g_{12}^* g_{21}^* e^{-3.6} & 0 \\ -g_{11}^* g_{21}^* e^{-11.8s} + g_{11}^* g_{21}^* e^{-5.2s} & g_{11}^* g_{22}^* e^{-5.2s} - g_{12}^* g_{21}^* e^{-11.8s} \end{bmatrix}}{g_{11}^* g_{22}^* e^{-3.6s} - g_{12}^* g_{21}^* e^{-10.2s}} \tag{91}$$

Note that this design requires much more complex control action

$$u = DC\epsilon \tag{92}$$

than the simpler design extending only properties two and three for this example, i.e.,

$$G_F = \begin{bmatrix} g_{11}^* & g_{12}^* \\ g_{21}^* e^{-6.6s} & g_{22}^* \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & e^{-1.6s} \end{bmatrix} \tag{93}$$

In addition this more complex design loses the predictive property two when one of the inputs hits a constraint, whereas the simpler design, Eq. 93, does not.

Obviously other, more general designs are possible, but it remains to be shown that the additional complexity of any of these yields significant practical advantages.

Systems With Right Half-Plane Zeroes

Although the GMDC is designed for compensation of time delays, the close similarity between time delays and right half-plane zeroes make it possible to augment the compensator design to handle cases where RHP zeroes arise. Suppose $G(s)$ has p RHP zeroes at locations $z_1, z_2, \ldots, z_p$. Then following suggestions by Frank (1974) and Holt and Morari (1985a), one should place a pole symmetric with each RHP zero for best performance. To accomplish this within the context of our generalized multidelay compensator we must design G_F so that both G_F^{-1} and G_F are stable. This requires augmentation of G_F such that

$$G_F = (G_F)_{\text{time delay}} \cdot \left[\frac{(s+z_1)(s+z_2)\cdots(s+z_p)}{(-s+z_1)(-s+z_2)\cdots(-s+z_p)}\right] \tag{94}$$

[Note that since $(G_F)_{\text{time delay}}$ contains RHP zeroes at $z_1, z_2, \ldots, z_p$, G_F in Eq. 94 is stable.] This augmentation means that properties one and three of the GMDC are not changed, but the predictive property two is modified so that for set point changes

$$\bar{f}_i(s) = \bar{y}_i(s)e^{+\theta_i s}\left[\frac{(s+z_1)(s+z_2)\cdots(s+z_p)}{(-s+z_1)(-s+z_2)\cdots(-s+z_p)}\right] \tag{95}$$

This means that each element, f_i, of the feedback vector is a forecast of y_i, a time delay θ_i plus a lead-lag into the future. Note that if we approximate each of the RHP zeroes by a first-order Padé approximation, i.e.,

$$\frac{-s+z_k}{s+z_k} \cong e^{-2/z_k s} \tag{96}$$

the RHP zeroes become time delays and Eq. 95 becomes

$$f_i(s) = y_i(s)\exp\left[+\left(\theta_i + \sum_k 2/z_k\right)s\right] \tag{97}$$

Thus the similarity between the treatment of time delays and RHP zeroes becomes clear.

Let us illustrate the procedure with an example.

Example 7

Consider the following system:

$$G(s) = \begin{bmatrix} \dfrac{(-s+1)e^{-2s}}{(s^2+1.5s+1)} & \dfrac{0.5(-s+1)e^{-4s}}{(2s+1)(3s+1)} \\ \dfrac{0.33(-s+1)e^{-6s}}{(4s+1)(5s+1)} & \dfrac{(-s+1)e^{-3s}}{(4s^2+6s+1)} \end{bmatrix} \tag{98}$$

which clearly has a right half-plane transmission zero at $s = +1$. A compensator design that deals only with the time delays, and extends properties two and three is:

$$D(s) = I \tag{99}$$

$$G_F(s)_{\text{time delay}} = \begin{bmatrix} \dfrac{(-s+1)}{(s^2+1.5s+1)} & \dfrac{(-s+1)e^{-2s}}{(2s+1)(3s+1)} \\ \dfrac{0.33(-s+1)e^{-3s}}{(4s+1)(5s+1)} & \dfrac{(-s+1)}{(4s^2+6s+1)} \end{bmatrix} \tag{100}$$

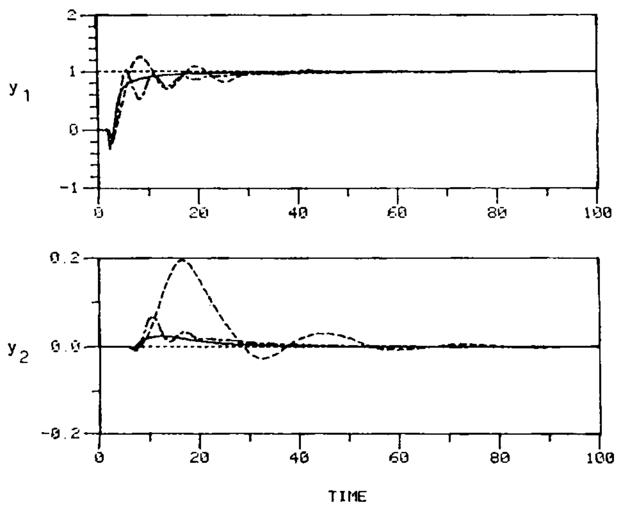

Figure 10. Response of example 7 system to a step of +1.0 in set point of y_1.

...... set points; ---- PI control only; _ _ . _ PI with standard GMDC designed to extend properties 2 and 3; ____ PI with GMDC augmented for RHP zero. Tuning parameters given in Table 1.

This compensator does allow significant improvement in the servo response, as shown below, but further improvement is possible via augmentation of this $G_F(s)$ (as described above) for the RHP zero.

The augmented $G_F(s)$ is:

$$G_F(s) = (G_F(s))_{\text{time delay}} \cdot \frac{(s+1)}{(-s+1)}$$

$$= \begin{bmatrix} \dfrac{(s+1)}{(s^2+1.5s+1)} & \dfrac{0.5(s+1)e^{-2s}}{(2s+1)(3s+1)} \\ \dfrac{0.33(s+1)e^{-3s}}{(4s+1)(5s+1)} & \dfrac{(s+1)}{(4s^2+6s+1)} \end{bmatrix} \tag{101}$$

$D(s)$ is unchanged.

The performance of these two compensator designs is compared in Figure 10 with that of conventional PI-only control for a unit step in the setpoint of y_1. In this figure the dashed curves are conventional PI. The dot-dash curves show the improvement with addition of the compensator based only on delay considerations. The solid curves illustrate the enhanced performance with the augmented compensator.

It should be noted that wrong-way or inverse response resulting from a combination of time delays often resembles polynomial RHP zeroes. This case is handled automatically by the GMDC applied to the general time delay model as shown in an earlier section.

Summary of Design Procedure

The design procedure for the generalized multidelay compensator (GMDC) is summarized in Table 2 for both simple and general time delay models. Note that the design depends on the time delay structure of the problem:

1. When all the delays are the same in a given row of $G(s)$ and this is true for each row, then all three properties are simultaneously extended by designing for property one.

2. When the rearrangement test is satisfied, then properties two and three are simultaneously extended by designing for property two. In this case the lower response limit is always achievable with sufficiently high gains.

3. When the rearrangement test is not satisfied, one has two options:

(a) If one wishes decoupled control and is willing to be satisfied by the decoupling response limit, then one can add time delays to some of the inputs through $\boldsymbol{D}$ so that $\boldsymbol{GD}$ passes the rearrangement test. In this case the best performance achievable at high controller gain is the decoupling response limit and properties two and three are both satisfied for $\boldsymbol{GD}$.

(b) If one wishes to sacrifice decoupled control to achieve more rapid response for some of the more important outputs, then property two can be sacrificed for some of the less critical outputs.

The design equations for each of these options have already been described.

As indicated in the previous section, in rare instances where right half-plane polynomial zeroes arise in addition to time delays, the compensator may be augmented to deal with this case. Complex time delay expressions that produce wrong-way behavior similar to RHP zeroes are already treated by applying the GMDC to the general time delay model.

Conclusion

A new generalized multidelay compensator (GMDC) has been developed that can extend one or more of the properties of the SISO Smith predictor using the structure proposed by Ogunnaike and Ray. Multidelay compensators designed on the basis of output prediction and process factorization (properties two and three) are found to provide generally better performance than those based on removing time delays from the characteristic equation (property one). In the special case when all the delays in a given row of the transfer function matrix are identical, all three properties are satisfied by the same compensator design. The compensator provides both time delay and interaction compensation and is shown to yield excellent performance on test examples such as distillation column control. Controller tuning through conventional (e.g., PI) controllers may be used to achieve a proper balance between performance and robustness. The compensator has the advantage that even in the presence of manipulated variable constraints, the feedback controller receives forecasts of each output from the compensator. This may explain the extremely good performance of the compensator even when controller constraints are applied.

When the GMDC is compared to other high-performance compensators (e.g., internal model control [IMC] of Garcia and Morari, 1984, several important differences are apparent. The IMC control structure attempts to directly invert the invertible part of the process, which is then multiplied by a filter. It is through design of this filter that the IMC designer shapes the final closed-loop response. With the GMDC structure, the design consists of a feedback loop involving conventional controllers $\boldsymbol{C}(s)$ and predictor $\boldsymbol{G}_F(s) - \tilde{\boldsymbol{G}}(s)$. For both IMC and GMDC the designer must select $\boldsymbol{G}_F(s)$ [called $\boldsymbol{G}^-(s)$ in IMC notation]. With GMDC $\boldsymbol{G}_F(s)$ must be causal [and if the design extends property three $\boldsymbol{G}_F(s)^{-1}$ it will be causal as well], whereas in IMC it is $\boldsymbol{G}_F(s)^{-1}$ that must be causal. With GMDC, as demonstrated repeatedly in this paper, the design procedure for selection of $\boldsymbol{G}_F(s)$ is well specified and can be applied almost automatically. With GMDC one must next tune the conventional single-loop controllers in the $\boldsymbol{C}(s)$ block. With IMC, one is next faced with a choice of $\boldsymbol{G}_F^{-1}$ and a filter design. However, when the outputs of GMDC conventional controllers, $\boldsymbol{C}(s)$, hit constraints, the predictive properties are preserved. By contrast, should one of the outputs from the IMC controller block hit a constraint, all claims to having a filtered inverse are lost. If this is to be preserved, the IMC filter must be selected such that the constraints are never hit, which is not only difficult to do, but often leads to sluggish performance in tightly constrained systems. Thus, in comparison to the IMC controller the GMDC has a simpler implementation structure, provides for simpler tuning through conventional controllers, and does not lose all its desirable properties when the control variables are clipped by constraints. Furthermore, the GMDC can achieve the same idealized control system performance at infinitely large controller gains, but can achieve most of this ideal performance at finite controller gains.

Remaining issues presently under study include characterizing the robustness properties of the GMDC, and experimental testing. These results will be reported in a later paper.

Acknowledgment

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Notation

c = controller for SISO system
$\boldsymbol{C}$ = controller for MIMO system
d = SISO disturbance
$\boldsymbol{d}$ = MIMO disturbance vector
$\boldsymbol{D}(s)$ = diagonal matrix of delays
$d_i = e^{-b_i s}$ = iith element of $\boldsymbol{D}(s)$
$\boldsymbol{E}(s)$ = diagonal matrix of delays
f = SISO feedback signal for Smith predictor
$\boldsymbol{f}$ = feedback vector for MIMO delay compensators
$g(s)$ = SISO plant = $g^*(s)e^{-\theta s}$
$g^*(s)$ = SISO plant without delay
$g_{ij}(s)$ = element ij of $\boldsymbol{G}(s)$
$g_{ij}^*(s)$ = element ij of $\boldsymbol{G}(s)$ without delays
$\boldsymbol{G}(s)$ = MIMO plant
$\boldsymbol{G}^*(s)$ = MIMO plant without delays
$\boldsymbol{G}_F(s)$ = MIMO modified model of plant (different delays) used in MIMO delay compensators
$\boldsymbol{G}^+$ = partition of $\boldsymbol{G}$ associated with $\boldsymbol{y}^+$
$\boldsymbol{G}^-$ = partition of $\boldsymbol{G}$ associated with $\boldsymbol{y}^-$
$\boldsymbol{G}_F^+$ = partition of $\boldsymbol{G}_F$ associated with $\boldsymbol{y}^+$
$\boldsymbol{G}_F^-$ = partition of $\boldsymbol{G}_F$ associated with $\boldsymbol{y}^-$
$\boldsymbol{I}$ = identity matrix
k_{ij} = steady state gain of $g_{ij}(s)$
$\boldsymbol{L}$ = lower response bound for $\boldsymbol{G}(s)$
$\boldsymbol{L}_d$ = lower response bound for $\boldsymbol{GD}(s)$
L_{ij} = number of terms in denominator of g_{ij}, general form
M_{ij} = number of terms in numerator of g_{ij}, general form
n = order of $\boldsymbol{G}(s)$
$p_{ijk}(s)$ = kth polynomial in numerator of $g_{ij}(s)$, general form
$P_{ij}(s)$ = numerator of $g_{ij}(s)$, general form
$q_{ij\ell}(s)$ = ℓth polynomial in denominator of $g_{ij}(s)$, general form
$Q_{ij}(s)$ = denominator of $g_{ij}(s)$, general form
r_j = largest prediction in jth column of $\boldsymbol{G}^{-1}(s)$
$\boldsymbol{R}(s)$ = diagonal decoupling performance limit for $\boldsymbol{G}(s)$
$\boldsymbol{R}_d(s)$ = diagonal decoupling performance limit for $\boldsymbol{GD}(s)$
$\boldsymbol{T}(s)$ = test matrix used in analysis of general form $\boldsymbol{G}(s)$ plants
t^*ij = the part of element ij of $\boldsymbol{T}(s)$ without delay
$u(s)$ = SISO manipulated variable

$u(s)$ = MIMO vector of manipulated variables
$xi(t)$ = state i of realization of compensator
$y(s)$ = SISO plant output
$\mathbf{y}(s)$ = MIMO vector of plant outputs
$\mathbf{y}^+(s)$ = partition of $\mathbf{y}(s)$; the outputs whose response is to be favored
$\mathbf{y}^-(s)$ = partition of $\mathbf{y}(s)$; the outputs whose response is to be sacrificed
y_{set} = SISO setpoint signal
$\mathbf{y}_{set}$ = MIMO vector of setpoint signals
z_i = ith component of compensator; or location of ith RHP zero

Greek letters

α_{ij} = delay associated with $g_{ij}(s)$, simple form
α_{ijk} = delay associated with $P_{ijk}(s)$, general form
$\tilde{\alpha}_{ijk}$ = shortest delay in numerator of $g_{ij}(s)$, general form
β_{ij} = delay associated with element (ij) of $\mathbf{GD}(s)$
$\beta_{ij\ell}$ = delay associated with $q_{ij\ell}(s)$ general form
δ_i = delay in element (ii) of $\mathbf{D}(s)$
θ = time delay for the SISO plant $g(s)$
θ_i = shortest delay in row i of $\mathbf{G}(s)$ or shortest delay in row i of $\mathbf{GD}(s)$
$\Theta(s)$ = diag $[e^{+\theta_1 s}e^{+\theta_2 s}\ldots e^{+\theta_n s}]$
τ_{ij} = delay associated with element (ij) of $\mathbf{G}_F(s)$, simple form

Appendix

Design of $D(s)$ and $G_F(s)$ for simultaneous extension of properties two and three

We assume that $\mathbf{G}(s)$ fails the rearrangement test. [If $\mathbf{G}(s)$ passes the test, then extension of property two also extends property three.] The objective is to select $\mathbf{D}(s)$ such that the apparent system $\mathbf{GD}(s)$ does pass the rearrangement test. Then $\mathbf{G}_F(s)$ may be designed for extension of property two for $\mathbf{GD}(s)$ and property three will be extended as well.

From theorems 4 and 5, $\mathbf{R} = \mathbf{R}_d = \mathbf{L}_d$ where

$$\mathbf{G}_F = \mathbf{R}^{-1}\mathbf{G}(s)\mathbf{D}(s) \tag{A1}$$

Here $\mathbf{R}(s)$ and $\mathbf{G}(s)$ are known and $\mathbf{D}(s)$ is required to be of the form $\mathbf{D}$ = diag $(-\delta_i s)$ with the smallest possible values of the δ_i's that still remove all predictions from the righthand side of Eq. A1. This means δ_i is equal in magnitude to the largest prediction in column i of the product $\mathbf{R}^{-1}(s)\mathbf{G}(s)$ · δ_i is zero if there is no prediction. Thus we have:

$$\delta_i = \max_j\ (0, \gamma_{ji}) \tag{A2}$$

where

γ_{ji} = largest prediction in element (ji) of $\mathbf{R}^{-1}(s)\mathbf{G}(s)$
$\mathbf{R}(s)$ = decoupling performance limit for $\mathbf{G}(s)$

The $\mathbf{G}_F$ is found from Eq. A1.

Treatment of general time delay models

The design of compensators for systems with elements of the general form of Eqs. 68–69 is based on formation of a test matrix $\mathbf{T}(s)$. The analysis is performed on this much simpler test matrix and the design based on $\mathbf{T}(s)$ is applied to the original $\mathbf{G}(s)$. The success of this approach is a consequence of theorem 6, which is stated below.

Theorem 6

Given: A $\mathbf{G}(s)$ matrix with elements of the form of Eqs. 68–69 and with the following properties:

1. $\mathbf{G}(s)$ is nonsingular.
2. Those terms in the determinant of $\mathbf{G}(s)$ with the shortest delays do not cancel exactly.
3. For each cofactor of $\mathbf{G}(s)$ those terms with the shortest delays contributing to that cofactor do not cancel exactly.

Let

$$\tilde{\alpha}_{ij} = \min_k \alpha_{ijk} \quad V_i, V_k \tag{A3}$$

$$\mathbf{T}(s) = \begin{bmatrix} t^*_{11}e^{-\tilde{\alpha}_{11}} & \cdots & t^*_{1n}e^{-\tilde{\alpha}_{1n}} \\ \vdots & & \\ t^*_{n1}e^{-\tilde{\alpha}_{21}} & \cdots & t^*_{nn}e^{-\tilde{\alpha}_{nn}} \end{bmatrix} \tag{A4}$$

where t^*_{ij} is a parameter that is zero if $G_{ij}(s) \equiv 0$.

Then the locations and magnitudes of all predictions occuring in $[\mathbf{G}(s)]^{-1}$ are identically matched by the locations and magnitudes of the predictions occurring in $[\mathbf{T}(s)]^{-1}$. In other words, the predictions occurring in $\mathbf{G}(s)^{-1}$ are only dependent on the shortest delay in the numerator of each element of $\mathbf{G}(s)$ (denoted here by $\tilde{\alpha}_{ij}$).

Proof: See Jerome (1986).

Design procedures for general models

The design procedures for extension of property one and two to $\mathbf{G}(s)$ systems of general form are described in the text. Here the extension of properties two and three or three alone are described.

Simultaneous Extension of Properties Two and Three

Assuming $\mathbf{G}(s)$ satisfies the requirement of theorem 6, the $\mathbf{T}(s)$ test matrix may be formed (as described in the text) and the rearrangement test applied to $\mathbf{T}(s)$. If $\mathbf{T}(s)$ passes the rearrangement test, then $\mathbf{G}(s)$ will also, since this test only depends on the distribution and magnitude of predictions in $\mathbf{T}(s)^{-1}$, which by theorem 6 is the same as in $\mathbf{G}(s)^{-1}$. So the decoupling and lower response bounds will be identical for $\mathbf{G}(s)$. Conversely, if $\mathbf{T}(s)$ fails the test, $\mathbf{G}(s)$ will also fail the test and the decoupling and lower response bounds will differ. (In the rare cases where theorem 6 cannot be applied because of cancelations, the decoupling and lower response bounds must be calculated directly from their definitions.) In any event, all $\mathbf{G}(s)$ will fall into one of two categories:

Case 1. $\mathbf{T}(s)$ Passes the Rearrangement Test.

For these $\mathbf{G}(s)$ systems, direct extension of property two (as described in the text) will also extend property three. These are the systems for which dynamic decoupling is optimal.

Case 2. $\mathbf{T}(s)$ Fails the Rearrangement Test.

For these systems a $\mathbf{D}(s)$ matrix, $\mathbf{D}(s) \neq \mathbf{I}$, is required. The procedure for determination of the delays in $\mathbf{D}(s)$ is analogous to the procedure for simple systems (see the first section of this appendix) except that the analysis is performed using the $\mathbf{T}(s)$ matrix instead of the $\mathbf{G}(s)$ matrix to make the calculations less burdensome.

One calculates the decoupling response bound $\mathbf{R}(s)$ based on $\mathbf{T}(s)$ and writes the analog of the righthand side of Eq. A1:

$$\mathbf{R}^{-1}(s)\mathbf{T}(s)\mathbf{D}(s) \tag{A5}$$

Here $\mathbf{R}^{-1}(s)$ and $\mathbf{T}(s)$ are known. $\mathbf{D}(s)$ is of the form of Eq. A2 and again the δ_i's are chosen to be as small as possible while still eliminating all predictions from Eq. A5. That is:

$$\delta_i = \max_j (0, \gamma_{ji}) \tag{A6}$$

where

γ_{ji} = largest prediction in element (j, i) of $\boldsymbol{R}^{-1}(s)\boldsymbol{T}(s)$ [and equivalently $\boldsymbol{R}^{-1}\boldsymbol{G}(s)$]

$\boldsymbol{R}(s)$ = decoupling response bound for $\boldsymbol{T}(s)$ [and equivalently $\boldsymbol{G}(s)$].

This defines $\boldsymbol{D}(s)$. For $\boldsymbol{G}_F(s)$ equation A1 is still used.

Extension of Property Three Only

Just as for systems described by the simple model, when a system of general form fails the rearrangement test, it is no longer in general optimal to dynamically decouple. Instead it may be useful to favor the response of some outputs at the expense of the others by sacrificing property two for the less important outputs.

The procedure for design of compensators to do this for systems of general form is very similar to that described in the text for simple-form systems. The only difference is that the $\boldsymbol{T}(s)$ matrix is used in place of $\boldsymbol{G}(s)$ in formulating the rearrangement test.

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